Modelling flotation with sedimentation by a system of conservation laws with discontinuous flux

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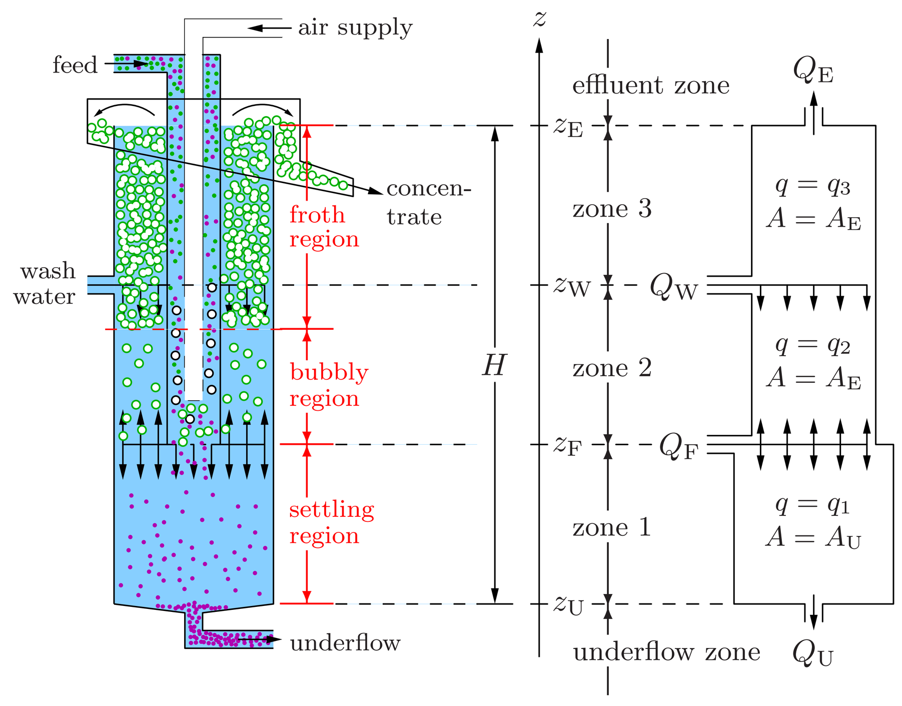
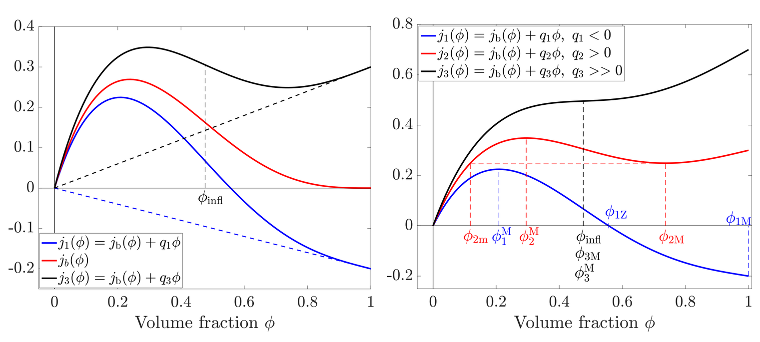
1. Three-phase flow of solids, gas (bubbles or aggregates) and fluid

Governing partial differential equations (Bürger et al. 2019a,b); ϕ: volume fraction of gas, φ: volume fraction of solids within the fluid, $A(z)$: cross-sectional area, $t > 0$: time, z: height.

$$A(z)\frac{\partial \phi}{\partial t} + \frac{\partial}{\partial z}\big(A(z)J(\phi, z, t)\big) = Q_{\mathrm{F}}(t)\phi_{\mathrm{F}}(t)\delta(z - z_{\mathrm{F}}),$$
$$A(z)\frac{\partial}{\partial t}\big((1-\phi)\varphi\big) - \frac{\partial}{\partial z}\big(A(z)F(\varphi, \phi, z, t)\big) = Q_{\mathrm{F}}(t)\phi_{\mathrm{s,F}}(t)\delta(z - z_{\mathrm{F}}) \tag{1}$$

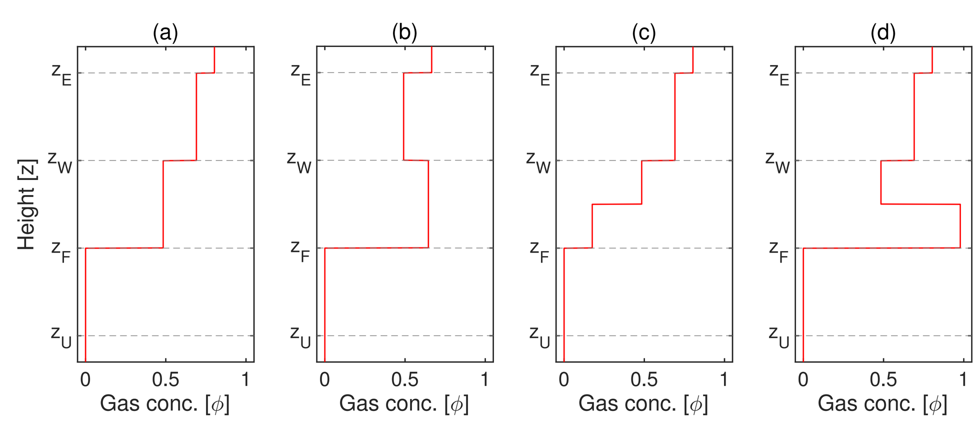




The fluxes J and F are discontinuous functions of z and incorporate batch drift and solid fluxes. Thus, (1) is a system of conservation laws with discontinuous flux and singular source terms.

2. Steady States and Operating Charts

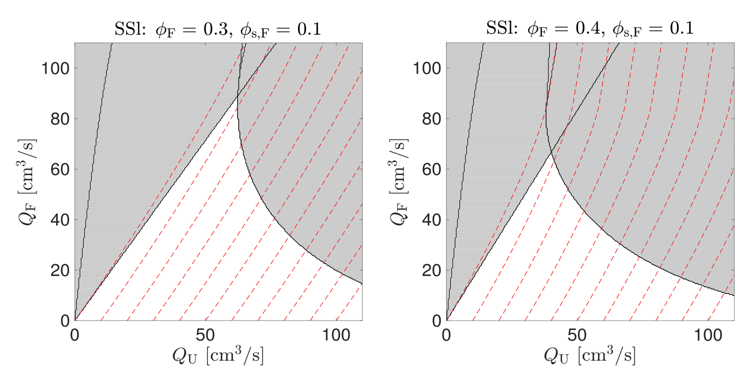
Stationary solutions, which have layers of different concentrations of bubbles and particles separated by discontinuities in concentration.



The different steady states depend on the values of the feed input volume fractions of the aggregates ϕ_{F} and the solids $\phi_{\mathrm{s,F}}$, and on the volumetric flow rates Q_{F}, Q_{U} and Q_{W}. *Desired steady states* have a high concentration of aggregates at the top (foam) and zero at the bottom. Three cases differ only in zone 2, where the solution can be constant low **(SSl)**, constant high **(SSh)**, or have a discontinuity separating these two values **(SSd)**.

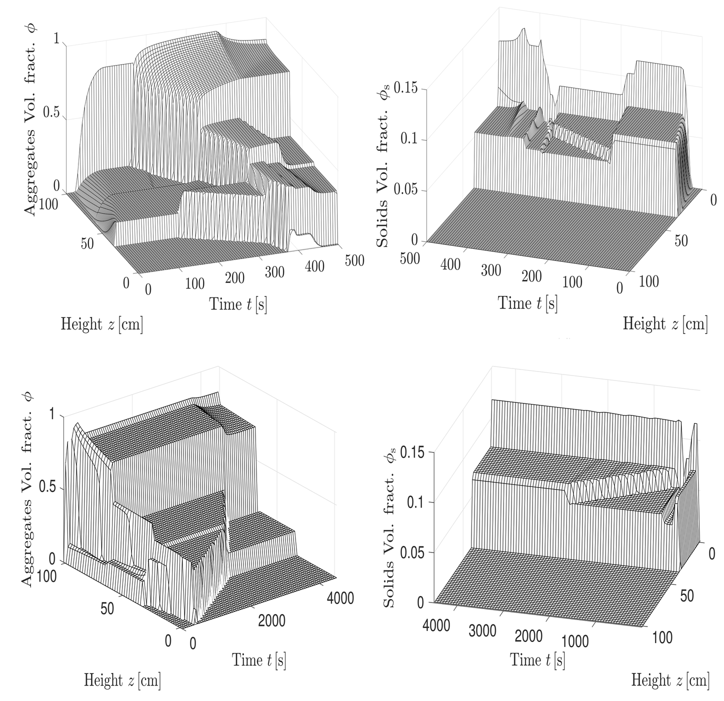
$$\phi_{\mathrm{SSl}}(z) := \begin{cases} \phi_{\mathrm{E}} = A_{\mathrm{E}}j_3(\phi_3)/Q_{\mathrm{E}} \geq \phi_{3\mathrm{M}} & \text{in the effluent zone,} \\ \phi_3 = \phi_{3\mathrm{M}} \geq \phi_2 & \text{in zone 3,} \\ \phi_2 \in [\phi_{2\mathrm{m}}, \phi_2^{\mathrm{M}}] & \text{in zone 2,} \\ 0 & \text{in zone 1 and underflow,} \end{cases}$$

Industrially relevant steady states and operating charts: The white region in the Figures show the possible values for $(Q_{\mathrm{U}}, Q_{\mathrm{F}})$. In each such point, there is unique value of Q_{W}.

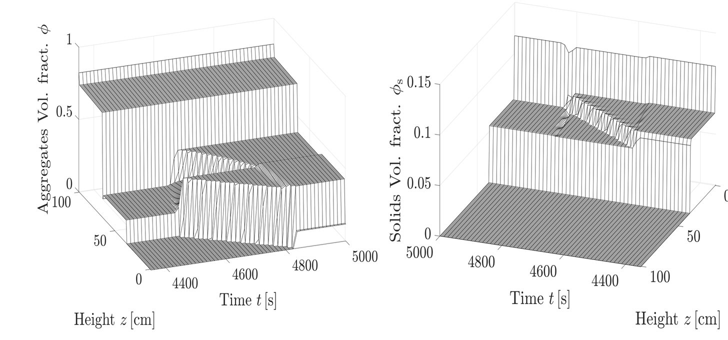


3. Numerical Results

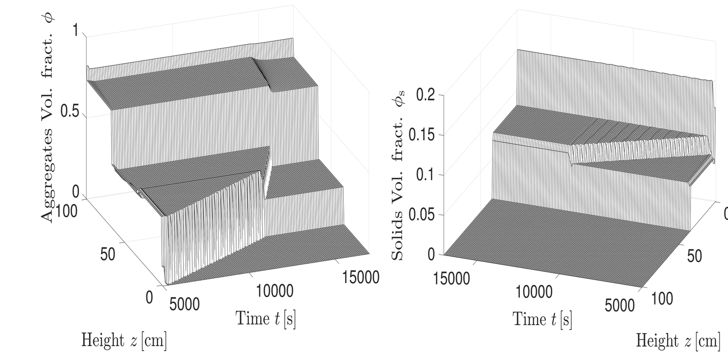
Example 1: We start from a tank filled only with fluid at time $t = 0\,\mathrm{s}$, where we start pumping aggregates, solids, fluid and wash water, with $\phi_{\mathrm{F}} = 0.3$ and $\phi_{\mathrm{s,F}} = 0.1$. A first steady state arises after about $t = 100\,\mathrm{s}$ with a low concentration of aggregates, then we 'close' the top of the tank at $t = 150\,\mathrm{s}$. The aggregates interact with the solid phase in zone 1 and leave through the underflow outlet. At $t = 350\,\mathrm{s}$, the top of the column is opened and a desired steady state of type SSl is reached after $t = 4500\,\mathrm{s}$.



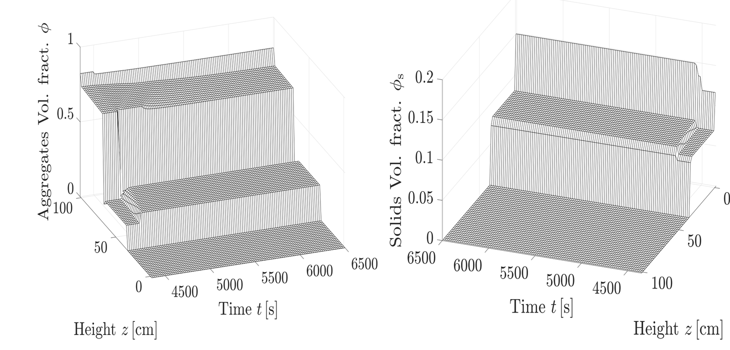
Once the system is in steady state, we change, at $t = 4500\,\mathrm{s}$, the feed volume fraction of aggregates from $\phi_{\mathrm{F}} = 0.3$ to 0.4, and simulate the reaction of the system. In the corresponding operating chart, the point is no longer in the white region; and no steady state of type SSl is feasible.



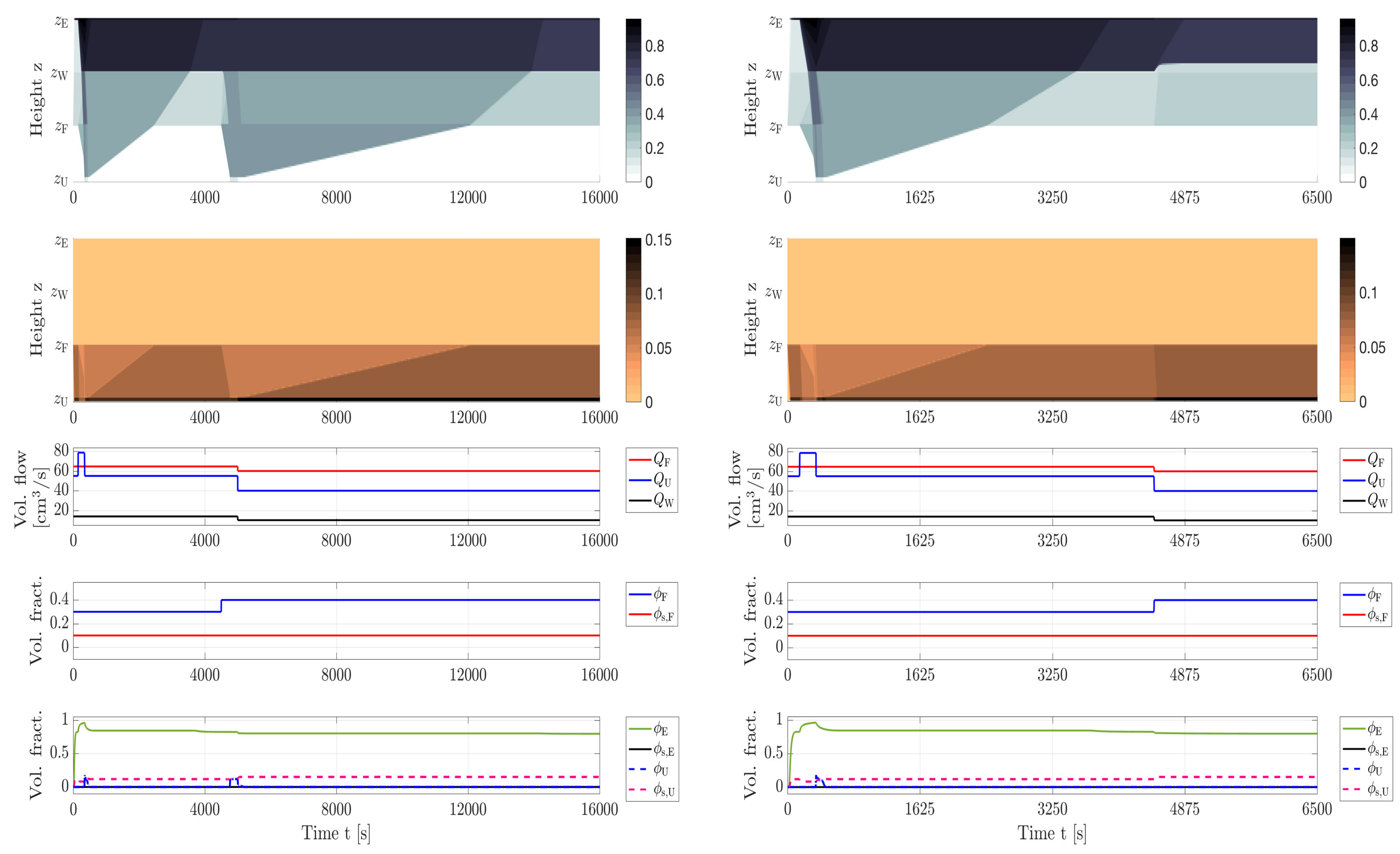
Once this new steady state is reached, we change, at $t = 5000\,\mathrm{s}$, the volumetric flows so that the new pointlies inside the white region of the operating chart (right). The Figure shows that a second steady state of type SSl is slowly reached after $t = 16000\,\mathrm{s}$.



Example 2: We let the simulation run until $t = 4500\,\mathrm{s}$ when the feed volume fraction ϕ_{F} made a step increase from 0.3 to 0.4 (as in Example 1). Instead of waiting with a control action to $t = 5000\,\mathrm{s}$, we now make the control action directly at $t = 4500\,\mathrm{s}$. A steady state of type SSl is quickly reached at about $t = 6500\,\mathrm{s}$.



The dynamics of the entire simulation for Example 1 and 2 can be found in the figures:



4. References and Acknowledgements

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