8th CI²MA Focus Seminar
"Numerical Analysis of PDEs: Theory, Methods and Applications"
Supported by Conicyt Project Anillo ACT1118 (ANANUM)

January 16, 2014
Auditorio Alamiro Robledo
Facultad de Ciencias Físicas y Matemáticas, Universidad de Concepción

Organizers: Raimund Bürger and Rommel Bustinza

Programme

14.30 **Mostafa Bendahmane** (Université Bordeaux Segalen, Francia):
Mathematical analysis and numerical approximation of a coupled system modeling cardiac electromechanics

15.00 **Michel Langlais** (Université Bordeaux Segalen, Francia):
A multiscale problem for plant pathogen dispersal

15.30 **Carlos D. Acosta** (Universidad Nacional de Colombia, Sede Manizales):
A numerical approach to time-fractional non linear convection-diffusion problems

16.00 **Pep Mulet** (Universidad de Valencia, España):
Weights design for maximal order WENO schemes and some techniques for improving the resolution of finite difference WENO schemes

16.30 **COFFEE BREAK**

17.00 **Gabriel N. Gatica** (Universidad de Concepción):
New developments on the coupling of mixed-FEM and BEM for the three-dimensional Stokes problem

17.30 **Ricardo Oyarzúa** (Universidad del Bío-Bío, Concepción):
A mixed-FEM for the coupling of Navier-Stokes and Darcy equations

18.00 **Giordano Tierra** (Charles University, Prague, Czech Republic):
Numerical methods for solving the Cahn-Hilliard equation and its applicability to two-phase flows with different densities

18.30 **Ariel Lombardi** (Universidad de Buenos Aires, Argentina):
Results for Nédélec edge elements on anisotropic tetrahedral meshes

19.00 **Rommel Bustinza** (Universidad de Concepción):
On the LDG method for the Helmholtz problem: priori and a posteriori error analyses

20.30 **Seminar Dinner**

Practical information

Seminar participants who would like to join dinner should register with CI²MA secretary:

Ms Angelina Fritz, CI²MA
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Abstracts

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MATHEMATICAL ANALYSIS AND NUMERICAL APPROXIMATION OF A COUPLED SYSTEM MODELING CARDIAC ELECTROMECHANICS

MOSTAFA BENDAHMANE

Abstract. This talk is concerned with the mathematical analysis of a coupled elliptic-parabolic system modeling the interaction between the propagation of electric potential and subsequent deformation of the cardiac tissue. The problem consists in a reaction-diffusion system governing the dynamics of ionic quantities, intra and extra-cellular potentials, and the elasticity equations are adopted to describe the motion of an incompressible material. The coupling between muscle contraction, biochemical reactions and electric activity is introduced with a so-called active strain decomposition framework, where the material gradient of deformation is split into an active (electrophysiology-dependent) part and an elastic (passive) one. We prove existence of weak solutions to the underlying coupled reaction-diffusion system and uniqueness of regular solutions. A finite element formulation is also introduced, for which we establish existence of discrete solutions and show convergence to a weak solution of the original problem. We close with a numerical example illustrating the behavior of the method and some features of the model.

This talk is based on recent joint work with B. Andreianov (Franche-comte, France), A. Quarteroni and R. Ruiz (EPFL Lausanne, Swiss).

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A MULTISCALE PROBLEM FOR PLANT PATHOGEN DISPERSAL

MICHEL LANGLAIS

Abstract. The spatial spread of an airborne pathogen within an anthropized crop plot depends on many factors acting at contrasted spatio-temporal scales: plant, plot or landscape scales. This is of paramount importance for vine and powdery mildew, one of its airborne pathogen. We shall first review a preliminary complex discrete model at the plant scale and an aggregated ODE system capturing the flavor of the local dynamics of the discrete one, with respect to available data. Then we aimed at developing a coupled PDEs-ODEs model for plant-pathogen interactions at the plot scale level in order to assess the effects of various host heterogeneities on the epidemic spread. Taking into account natural spatial periodicity of rows generated by anthropized cultural practices allows to use homogenization techniques to derive a further simplified model.
This contribution is based on recent joint work with JB. Burie (IMB, Bordeaux), A. Calonnec (INRA, Bordeaux), A. Ducrot (IMB, Bordeaux) and Y. Mammeri (LAMFA, Amiens).

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A NUMERICAL APPROACH TO TIME-FRACTIONAL NON LINEAR CONVECTION-DIFFUSION PROBLEMS

CARLOS D. ACOSTA

Abstract. We are interested in numerical strategies for problems of the form

$$\begin{cases} u_t^\alpha + bu_x = A(u)_{xx}, & (x,t) \in Q_T \\ u(x,0) = u_0(x), & x \in \mathbb{R} \end{cases},$$

where u_t^α denotes Caputo's Fractional derivative of order α $(0 < \alpha < 1)$, $Q_T = \mathbb{R} \times (0,T)$, $b > 0$ and $A(u)$ is an integrated diffusion coefficient of the form

$$A(u) = \int_0^u a(s)ds, \quad \text{with } a(u) \geq 0, \quad \text{and } A(u) \in L^1(\mathbb{R}) \cap L^\infty(\mathbb{R}).$$

A commonly used discretization of the fractional derivative is extended to this case. The spatial derivatives are computed with finite differences. The resulting method is proved to be consistent, conservative, stable and with spatial and temporal regularity. Some illustrative numerical results are included.

This contribution is based on recent joint work with P. Amador (Manizales, Colombia) and C. Mejía (Medellín, Colombia).

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WEIGHTS DESIGN FOR MAXIMAL ORDER WENO SCHEMES AND SOME TECHNIQUES FOR IMPROVING THE RESOLUTION OF FINITE DIFFERENCE WENO SCHEMES

PEP MULET

Abstract. Weighted essentially non-oscillatory (WENO) finite difference schemes, developed by Liu, Osher and Chan in [5] and improved by Jiang and Shu in [4], are one of the most popular methods to approximate the solutions of hyperbolic equations. But these schemes fail to provide maximal order accuracy near smooth extrema, where the first derivative of the solution becomes zero.

Some authors have addressed this problem with different weight designs. In this paper we focus on the weights proposed by Yamaleev and Carpenter in [6]. They propose new weights to provide faster weight convergence than those presented in [3] and deduce some constraints on the weights parameters to guarantee that the WENO scheme has maximal order for sufficiently smooth solutions with an arbitrary number of vanishing derivatives.

We analyze the scheme with the weights proposed in [6] and prove that near discontinuities it achieves worse orders than classical WENO schemes. In order to solve these accuracy problems, we define new weights, based on those proposed in [6], and get some constraints on the weights parameters to guarantee maximal order accuracy for the resulting schemes.

Component-wise finite difference WENO schemes are adequate when the full characteristic structure of a hyperbolic system of conservations laws cannot be computed in closed form. The drawback of these schemes is that they prescribe excessive diffusion and they may present spurious oscillations near shocks. In this work we propose to use a flux-splitting that prescribes less numerical viscosity for component-wise finite difference WENO schemes. We compare this technique with others to alleviate the diffusion and oscillatory behavior of the solutions obtained with component-wise finite difference WENO methods.

This contribution is based on recent joint work [1, 2] with F. Aràndiga and M.C. Martí.

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NEW DEVELOPMENTS ON THE COUPLING OF MIXED-FEM AND BEM FOR THE THREE-DIMENSIONAL STOKES PROBLEM

GABRIEL N. GATICA, GEORGE C. HSIAO, SALIM MEDDAHI, AND FRANCISCO J. SAYAS

Abstract. In this paper we study the coupling of a dual-mixed variational formulation, in which the velocity, the pressure and the stress are the main unknowns, with the boundary integral equation method for the three dimensional Stokes problem. In particular, following a similar analysis given recently for the Laplacian, we are able to extend the classical Johnson & Nédélec procedure to the present case, without assuming any restrictive smoothness requirement on the coupling boundary, but only Lipschitz-continuity. More precisely, after using the incompressibility condition to eliminate the pressure, we consider the resulting velocity-stress approach with a Neumann boundary condition on an annular bounded domain, and couple the underlying equations with the single boundary integral equation arising from the application of the normal trace to the Green representation formula in the exterior unbounded region. As a result, we obtain a saddle point operator equation, which is then analyzed by the well-known Babuška-Brezzi theory. We prove the well-posedness of the continuous formulation, identifying previously the space of solutions of the associated homogeneous problem, and give explicit finite element and boundary element subspaces guaranteeing the stability of the respective Galerkin scheme. The Costabel & Han coupling procedure is also considered, and corresponding results are provided as well.

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A MIXED-FEM FOR THE COUPLING OF NAVIER-STOKES AND DARCY EQUATIONS

RICARDO OYARZÚA

Abstract. In this talk we introduce and analyze a mixed finite element method for the coupling of fluid flow with porous media flow. Flows are governed by the Navier-Stokes and Darcy equations, respectively, and the corresponding transmission conditions are given by mass conservation, balance of normal forces, and the Beavers-Joseph-Saffman law. We consider the standard mixed formulation in the Navier-Stokes domain and the dual-mixed one in the Darcy region, which yields the introduction of the trace of the porous medium pressure as a suitable Lagrange multiplier. We use a classical fixed point argument to prove existence and uniqueness of solution of the coupled problem under a smallness assumption on the data. The finite element subspaces defining the discrete formulation employ Bernardi-Raugel and Raviart-Thomas elements for the velocities, piecewise constants for the pressures, and continuous piecewise linear elements for the Lagrange multiplier. Similarly to the continuous case, we show stability and well-posedness of the discrete problem. In addition, the a priori error estimate for the associated Galerkin scheme is provided for small data. This contribution is based on recent joint work with Marco Discacciati from Laboratory of Computational Methods and Numerical Analysis, Department of Applied Mathematics III, Universitat Politècnica de Catalunya, Barcelona, Spain.

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NUMERICAL METHODS FOR SOLVING THE CAHN-HILLIARD EQUATION AND ITS APPLICABILITY TO TWO-PHASE FLOWS WITH DIFFERENT DENSITIES

GIORDANO TIERRA

Abstract. The Cahn-Hilliard model was originally introduced by Cahn and Hilliard [2] to describe the complicated phase separation and coarsening phenomena in the mixture of different fluids, solid or gas where only two different concentration phases can exist stably.

During the seminar, I will present different numerical schemes to approximate the Cahn-Hilliard model, showing the advantage and disadvantages of each scheme. In particular, I will focus on the study of the constraints on the physical and discrete parameters that can appear to assure the energy-stability, unique solvability and, in the case of nonlinear schemes, the convergence of Newton's method to the nonlinear schemes. Moreover, an adaptive time stepping algorithm will be presented and the behavior of the schemes will be compared through several computational experiments.

In the second part of the seminar, I will summarize the model presented in [1] that can be viewed as a thermodynamic consistent coupling between Navier-Stokes for fluids with different densities and the Cahn-Hilliard model. The ideas presented in [5] will be shown, where a splitting numerical scheme that segregates the fluid part from the phase field part in a linear an energy-stable way has been derived. Finally, some numerical simulations for this model will be presented to show the effectiveness of the proposed numerical scheme.

This contribution is based on recent joint work with F.Guillén-Gonzaléz (Sevilla, Spain).

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RESULTS FOR NÉDÉLEC EDGE ELEMENTS ON ANISOTROPIC TETRAHEDRAL MESHES

ARIEL L. LOMBARDI

Abstract. The first family of Nédélec's edge elements, introduced in [3], is a conforming family of finite elements in $H(\mathrm{curl})$. It is broadly used in the approximation of elliptic partial differential equations in mixed form, such as Maxwell equation and their associated eigenproblems. Anisotropic meshes appear naturally in applications when the solution presents boundary layers or edge singularities. This is the case when considering the time-harmonic Maxwell equations in a Lipschitz polyhedron with nonconvex edges or corners. The possibility of using anisotropic elements can make the design of such meshes easier, reduce the number of elements and take advantage of the best regularity properties of the solution. In fact, in many problems, the solutions have more regularity in the direction of the edges than transversally to them.

In this talk firstly we show that uniform interpolation error estimates for edge elements can be obtained on tetrahedral meshes under the maximum angle condition. This condition allows for arbitrarily anisotropic elements needed for the discretisation of elliptic problems in general polyhedra.

Secondly, for the tetrahedral meshes used on general polyhedra, we discuss a proof of the discrete compactness property (DCP) for edge elements of any order. The DCP was introduced by Kikuchi [1] for edge elements of lowest order on shape-regular meshes. It is a useful tool for the analysis of the approximation of Maxwell's equations, both for the source problem as well as for computing the eigenvalues (or resonant frequencies) on a bounded cavity. The numerical approximation of both problems, and so, the validity of the DCP, has been considered in different situations by several authors: Boffi, Buffa, Caorsi, Costabel, Dauge, Fernandez, Hiptmair, Monk, Nicaise, Raffetto and others.

In particular, the aspects mentioned before extend some results of Nicaise [4] and Buffa, Costabel and Dauge [2]. Precisely, we consider edge elements of any order, and we allow edge and corner refinements: our meshes are proposed in order to be able to adequately approximate the homogeneous Dirichlet problem for the Laplace operator with a right hand side in L^p for some $p \geq 2$.

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ON THE LDG METHOD FOR THE HELMHOLTZ PROBLEM: A PRIORI AND A POSTERIORI ERROR ANALYSES

ROMMEL BUSTINZA

Abstract. In this talk we give an overview of the main features of discontinuous Galerkin method when applied to problem coming from the continuum mechanics. In particular, we focus on the applicability of the local discontinuous Galerkin (LDG for short) method to solve a mixed value problems for the Helmholtz equation in a bounded polygonal domain in 2D. Under some assumptions on regularity of the solution of an adjoint problem, we prove that: (a) the corresponding indefinite discrete scheme is well posed; (b) there is convergence with the expected convergence rates as long as the mesh size h is small enough. We also present an a posteriori error estimator, which is derived applying a suitable Helmholtz decomposition and is reliable and efficient with constants that depend on the wave number. Next, we show some numerical experiments which illustrate the theoretical results proven in this work. We finish giving some conclusions and final comments.

This contribution is based on recent joint work with T. Barrios (Concepción) and V. Domínguez (Tudela, Spain).

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