S A N T I A G O N U M É R I C O III
Noveno Encuentro de Análisis Numérico de Ecuaciones Diferenciales Parciales
Departamento de Matemática, Pontificia Universidad Católica de Chile
SANTIAGO, CHILE, JUNIO 28 - 30, 2017

Stabilized finite element methods for ill-posed problems with conditional stability *

Erik Burman†

Abstract

The design and analysis of computational methods for partial differential equations typically relies heavily on the well-posedness of the system under study, both to prove that the finite dimensional system resulting from discretization is invertible and for error estimates on the discrete solution. In practice a large class of problems are indefinite, with stability properties that are not so easily exploited for the design of numerical methods, or even ill-posed, with only some conditional stability. In such cases typically standard finite element methods may fail: the discrete system may not be invertible and when it is, the solution may be inaccurate, even if the ill-posed problem admits a unique solution in a neighbour hood of the available data. The standard way to approach ill-posed problems is through regularisation of the continuous problem and then discretization of the resulting, well-posed, perturbed problem, this however leads to the need of balancing errors due to regularization and discretization.Here we will discuss a different approach based on a stabilized primal-dual finite element formulation, introduced in [1] for the approximation of indefinite elliptic problems. An analysis in the ill-posed case was proposed in [2, 3]. This method is based on the discretization of the ill-posed problem *without any regularization on the continuous level*, instead the discrete system is regularized using ideas from stabilized finite element methods. Sometimes these stabilizing terms coincide with typical Tikhonov regularizations, but in many cases the regularizations do not have an interpretation on the continuous level. We show that the discrete system is always invertible and that the approximate solution satisfies (conditional) a priori and a posteriori error estimates that match the approximation order of the finite element space and the conditional stability properties of the discrete solution. Data perturbations also enter the analysis in a natural way. Three different problems will be discussed to illustrate the theory: The elliptic Cauchy problem, where Dirichlet and Neumann data are set only on a subset of the boundary; an elliptic data assimilation problem where the solution is known in a subset of the bulk, but no boundary data is available; finally a parabolic problem with Dirichlet boundary conditions, where the solution is known in some space time cylinder, but the initial data is unknown [4].

*This work was partially supported by EPSRC project EP/J002313/1

†Department of Mathematics, University College London, email: e.burman@ucl.ac.uk.

Key words: ill-posed problems, data assimilation, stabilized finite element methods

Mathematics subject classifications (1991): 65N30, 35R25, 65N20

References

[1] E. Burman. *Stabilized finite element methods for nonsymmetric, noncoercive, and ill-posed problems.* Part I: Elliptic equations. *SIAM J. Sci. Comput.*, 35(6):A2752–A2780, 2013.

[2] E. Burman. *Error estimates for stabilized finite element methods applied to ill-posed problems. C. R. Math. Acad. Sci. Paris*, 352(7-8):655–659, 2014.

[3] E. Burman. *Stabilised finite element methods for ill-posed problems with conditional stability. Building bridges: connections and challenges in modern approaches to numerical partial differential equations*, 93127, Lect. Notes Comput. Sci. Eng., 114, Springer, [Cham], 2016., Dec. 2015.

[4] E. Burman and L. Oksanen. *Data assimilation for the heat equation using stabilized finite element methods. ArXiv e-prints*, Sept. 2016.