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Approximate Lax-Wendroff discontinuous Galerkin methods for hyperbolic conservation laws *

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Abstract
The Lax-Wendroff time discretization [4] is an alternative method to the popular total variation diminishing Runge-Kutta time discretization of discontinuous Galerkin schemes for the numerical solution of hyperbolic conservation laws. The resulting fully discrete schemes are known as LWDG and RKDG methods, respectively. Although LWDG methods are in general more compact and efficient than RKDG methods (cf., e.g., [2]) of comparable order of accuracy, the formulation of LWDG methods [3, 5, 6] involves the successive computation of exact flux derivatives. This procedure allows to construct schemes of arbitrary formal order of accuracy in space and time. A new approximation procedure, implemented in [7] for finite difference schemes, avoids the computation of exact flux derivatives. The resulting approximate LWDG schemes, addressed as ALDWG schemes, are easier to implement than their original LWDG versions. Numerical results for the scalar and system cases in one and two space dimensions indicate that ALWDG methods are more efficient in terms of error reduction per CPU time than LWDG methods of the same order of accuracy. Moreover, increasing the order of accuracy leads to substantial reductions of numerical error and gains in efficiency for solutions that vary smoothly. This contribution summarizes results of [1].

Key words: Discontinuous Galerkin scheme, Lax-Wendroff time discretization, systems of conservation laws

Mathematics subject classifications (1991): 76S05, 65M08, 65M60, 65M12

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